count of the meeting, in September last, at Vienna, of the German Mathematical Association. The *Bulletin* well maintains its position, and closes with its useful lists of new publications.

SOCIETIES AND ACADEMIES.

LONDON.

Mathematical Society, Dec. 13, 1894.—Major Macmahon, F.R.S., President, and subsequently Mr. A. E. H. Love, F.R.S., Vice-President, in the chair.—The following communications were made:—On Maxwell's law of partition of energy, by Mr. G. H. Bryan. In his recent report to the British Association, the author had shown that if a large number of dynamical systems of any kind be taken, all similar in every respect, it is always possible to distribute their coordinates and momenta so that the distribution shall remain permanent, and shall satisfy Maxwell's law of partition of energy. By this is meant that if the kinetic energy of each system be reduced to a sum of squares, the mean values of these squares are equal. But the author had doubted whether it was possible in any case to infer that the *time averages* of the squares forming the kinetic energy of a single system were equal. In the present paper the connection between *time averages*, and *averages taken over a large number of different systems*, is examined more fully by means of an artifice suggested by Prof. Boltzmann's paper "On the application of the determinantal relation to the theory of polyatomic gases" (published as an appendix to Mr. Bryan's report). Instead of a vessel containing gas (as taken by Prof. Boltzmann), any single dynamical system is taken whose coordinates and momenta return to their original values after a long time T. If the time be divided into an infinitely large number (n) of infinitely short intervals (i), we can derive a stationary distribution by taking n systems and starting them, the first at time 0, the second at time i, the third at time $2i$, and so on, giving every system the same coordinates and momenta at the time of starting it. At the end of the time T, we shall have the systems distributed according to a permanent or stationary law, and at any subsequent instant the mean value of any function of the coordinates and momenta for the different systems will be equal to the time average of the corresponding function for the original single system. If, however, we *start* with a number of systems distributed according to a permanent law, we *cannot pass back to the original single system unless we can show that the law of permanent distribution is unique.* Now in any simple test case, such as that afforded by rigid bodies movable about fixed points or particles moving in planes after the manner of a Lissajou's curve tracer, *a* stationary distribution exists satisfying Maxwell's Law of Partition, but other stationary distributions are possible which do not satisfy the law. Hence the author concludes that the time averages of the squares into which the kinetic energy of a single system can be divided, are *not* in general equal, at any rate independently of initial circumstances.—The Spherical Catenary, by Prof. A. G. Greenhill, F.R.S. The pseudo-elliptic integrals developed in the *Proc.* L.M.S. xxv. are applied in this paper to the construction of solvable cases of the spherical catenary, given by the relation

$$\psi = \int \frac{A\,dz}{(1 - z^2)\sqrt{Z}},$$

where

$$Z = (1 - z^2)(z - h)^2 - A^2,$$

connecting ψ the azimuth, and $z = \cos\theta$, where θ denotes the angular distance from the lowest point of the sphere, the tension being $w(z - h)$ (Clebsch, *Crelle*, 57). Putting

$$u = \int \frac{dz}{\sqrt{Z}},$$

and

$$\chi = \psi - pu = \int \frac{-p(1 - z^2) + A}{(1 - z^2)\sqrt{Z}}\,dz,$$

then χ can be identified with the standard form of the pseudo-elliptic integral of the third kind, of order μ,

$$I = \tfrac{1}{2}\int \frac{\rho(s - \sigma) - \mu\sqrt{(-\Sigma)}}{(s - \sigma)\sqrt{S}}\,ds,$$

where

$$S = 4s(s + x)^2 - \{(y + 1)s + xy\}^2$$

by putting

$$\chi = \frac{I}{\mu},\ p = \tfrac{1}{2}\left(\frac{M\rho}{\mu} + A\right), A = M(y + 1), h^2 = A^2 - 2y - 1,$$

where

$$M^2 = -\frac{y + 1}{2x},$$

and x, y are Halphen's functions, defined in his "Fonctions Elliptiques," I. p. 102. If $u = a$ when $z = 1$, and $u = b$ when $z = -1$, then $u = \frac{1}{2}(a - b)$ when $z = h$; a and b are each of the form $f\omega_3$, a fraction of the imaginary period ω_3. Also

$$M^2\wp(a + b) = -\tfrac{1}{6}(1 - h^2) - \tfrac{1}{4}A^2,$$

or $12\wp(a + b) = 8x - (y + 1)^2$, so that $\sigma = 0$, and

$$a + b = \frac{4\omega_3}{\mu}.$$

Thus, for instance, when

$$a + b = \omega_3,\ A = h^2 - 1,\ p = \tfrac{1}{2}A;$$

and the corresponding spherical catenary is given by

$$(1 - z^2)^{\frac{1}{2}}e^{\chi i} = \sqrt{\frac{h - 1}{2}}\sqrt{\{z^2 - (h + 1)z - 1\}}$$
$$+ i\sqrt{\frac{h + 1}{2}}\sqrt{\{-z^2 + (h - 1)z + 1\}}.$$

With

$$\mu = 3,\ a + b = \tfrac{4}{3}\omega_3,\ A^2 = h^2 - 1,\ p = {}^{1}_{3}A;$$

and

$$(1 - z^2)^{\frac{3}{2}}e^{3\chi i} = A(z^3 - hz^2 - 2z) + i(hz + 1)\sqrt{Z}.$$

With

$$\mu = 5,\ a + b = \tfrac{4}{5}\omega_3,$$

then $y = x = -c$, suppose;

$$M^2 = \frac{1 - c}{2c},\ p = \tfrac{1}{5}(3 - c)M;$$

and

$$(1 - z^2)^{\frac{5}{2}}e^{5\chi i} = Hz^5 + \ldots + H_5 + i(Lz^3 + \ldots + L_3)\sqrt{Z},$$

where

$$H = \frac{2 - 5c + c^2}{c}M,\ L = \frac{2 - c}{c}h, \text{ \&c.}$$

With

$$a + b = \frac{1}{2}\omega_3$$

the calculation is rather more complicated, as this case must be derived from $\mu = 8$; but the result is of the form

$$(1 - z^2)e^{2\chi i} = (Hz - H_1)\sqrt{(z_3 - z\,.\,z - z_0)}$$
$$+ i(Lz - L_1)\sqrt{(z - z_2\,.\,z - z_1)},$$

with

$$z_3 > z > z_2 > z_1 > z > z > z_0,$$
$$z_0,\ z_1,\ z_2,\ z_3$$

denoting the roots of the quartic $Z = 0$.—The Transformation of Elliptic Functions, by Prof. A. G. Greenhill, F.R.S. The function z_a, introduced by Prof. Klein in his paper on the transformation of elliptic functions (*Proc.* L.M.S. xi. p. 151), and developed in Klein and Fricke's "Modulfunctionen," ii. part v., is shown here to be connected with Halphen's γ function by the relation

$$\rho(-1)^a z_a = \lambda^{\overset{a^2}{n}}\gamma_a,$$

for a transformation of the nth order; and for an odd value of n,

$$\lambda^{2p+1} = \frac{\gamma_{\frac{n-2p+1}{2}}}{\gamma_{\frac{n+2p-1}{2}}},\quad p = 1, 2, 3 \ldots \frac{n - 1}{2};$$

in this manner the relation $\gamma_n = 0$ is satisfied.

The biquadratic relations satisfied by the function z_a are now derived from Halphen's formula

$$\gamma_{m+n}\gamma_{m-n} = \gamma_{m+1}\gamma_{m-1}\gamma_n^2 - \gamma_{n+1}\gamma_{n-1}\gamma_m^2$$

The relation $\gamma_n = 0$ is treated as the equation of a curve with coordinates x and y; and when x and y can be expressed as functions of a parameter c, the quantities γ_α and z_α can also be expressed as functions of this parameter. For instance, $\gamma_{13} = 0$ reduces to the quadratic in p

$$p^2 - (1 - c^2 - c^3)p - c(1 + c)^2 = 0$$

by means of the substitutions

$$x = y(1 - z), \quad y = z\left(1 - \frac{z}{p}\right), \quad z = c(p - 1);$$

and now in Klein's Modular Equation of the Thirteenth Order, given in the *Proc.* L.M.S. ix. p. 126, it is found that

$$\tau = \frac{1 - c - 4c^2 - c^3}{c(1 + c)} = \frac{1}{c} + \frac{1}{1 + c} - c - 3.$$

When one root τ_∞ of this modular equation is given by

$$\tau_\infty = \frac{13c(1 + c)}{1 - c - 4c^2 - c^3} = \frac{13}{\frac{1}{c} + \frac{1}{1 + c} - c - 3},$$

the remaining 13 roots, typified by τ_r, are given by

$$\tau_r = \frac{\left(1 + \sum \frac{z_{2\alpha}}{z_\alpha} \epsilon^{36\alpha^2 r}\right)^2}{\frac{1}{c} + \frac{1}{1 + c} - c - 3}, \quad \epsilon = e^{\frac{2\pi i}{13}},$$

where $\alpha = 1, 2, 3, 4, 5, 6$; $r = 0, 1, 2, \ldots 12$,

(Klein, *Math. Ann.* xvii. p. 567).

It has been shown in the *Proc.* L.M.S. xxv. p. 252, that

$$\lambda = -\frac{cx^{\frac{1}{3}}}{yz};$$

and now, putting

$$\rho = \sigma x^{\frac{1}{3}},$$

$$(\sigma z_1)^{13} = \frac{c}{x^4 yz}, \quad (\sigma z_2)^{13} = \frac{c^4}{x^3 y^4 z^4}, \quad (\sigma z_3)^{13} = \frac{x^3 c^9}{y^9 z^9},$$

$$(\sigma z_4)^{13} = \frac{z^{16} c^{16}}{xy^3}, \quad (\sigma z_5)^{13} = \frac{c^{25}}{x^4 y^{12} z^{12}}, \quad (\sigma z_6)^{13} = \frac{c^{36}}{x^{12} y^{23} z^{10} p},$$

all functions of c and $\sqrt{C}$, where

$$C = 1 + 4c + 6c^2 + 2c^3 + c^4 + 2c^5 + c^6.$$

The solution in the same manner of the Modular Equation of the Seventh Order has already been given in the *Proc.* L.M.S. xxv. p. 224; while the Fifth Order introduces Klein's icosahedron function. A similar procedure will serve for the Eleventh Order.—On certain definite theta-function integrals, by Prof. L. J. Rogers.—On a class of groups defined by congruences (second paper), by Prof. W. Burnside, F.R.S.—Electrical vibrations in condensing systems, by Dr. J. Larmor, F.R.S. It is only by the introduction of considerable capacity that the vibrations of electrical systems of simple geometrical form can assume a character at all simple and steady; for this reason it is of practical importance to be able to estimate the periods of vibrations in the dielectric plates of condensers. It is shown that the modes and periods for such a plate are precisely the same as those of the acoustical vibrations of a plate of air of the same form and the same law of thickness, enclosed on both faces, and also round its edge, by rigid walls, the only difference being that the velocity of electric propagation replaces the velocity of sound. For example, if the condenser is a spherical one, the periods of the free vibrations are equal to the time required for an electric pulse to travel round its circumference, divided by $(m^2 + m^{\frac{1}{2}}$, where m is any integer; and this result will also practically hold good for a condenser of this form which is not a complete sphere, but has a hole through it at the point opposite to the place where the inner coating is connected with the exciter. The periods for a flat condenser of uniform thickness correspond to the well-known ones of standing water-waves in a cylindrical vessel of the same form of contour. If a condenser is divided by cutting across its conducting coats, as is done in the ordinary guard-ring, the separate parts will vibrate without interfering with each others periods. Various other cases are treated; for example, the propagation of electric waves in a compound plate composed, say, of air above and a liquid below.—On the integration of Allegrét's integral, by Mr. A. E. Daniels.—On the complex number formed by two quaternary matrices, by Dr. G. G. Morrice.

Linnean Society, December 20, 1894.—Mr. C. B. Clarke, F.R.S., President, in the chair.—Mr. W. B. Hemsley exhibited a series of specimens and figures illustrating parasitism of *Loranthus aphyllus* and other plants, from the Herbarium, Kew.—Mr. J. E. Harting exhibited a specimen of a small Siberian warbler, *Phylloscopus superciliosus*, which had been obtained near Beverley, Yorkshire, in October last, and made some remarks on its haunts, habits, and migration, and upon the previous instances which had been noted of its accidental occurrence in the British Islands.—Mr. H. M. Bernard gave the substance of a paper on the spinning glands in *Phrynus*, not previously known, and described their position and their morphological importance in Arachnidan phylogen. The penis was described as a pair of rudimentary filamentous appendages of the genital segment, and consequently of importance as bearing further testimony to the view that the limbs on the abdomen of the ancestral form were not plates as in *Limulus*, but appendages like those on the thorax. The presence of these limbs explains the curious genital operculum of the *Pedipalpi*, which is not a primitive feature derived from Eurypterine ancestors, as some would maintain, but a purely secondary specialisation acquired within the Arachnid phylism.—A paper was then read by Mr. Percy Groom, entitled "Contributions to the knowledge of Monocotyledonous saprophytes," or plants which are dependent for their existence on the presence in the substratum of decaying organic matter. He observed that, like parasites, they may be divided into those which possess chlorophyll (*hemisaprophytes*) and those which have none (*holosaprophytes*). Hitherto very few experiments, he said, had been made on *hemisaprophytes*, and hence our acquaintance with them was largely speculative. The remarks which he had now to offer referred almost entirely to *holosaprophytes*, or at least to plants with very little trace of chlorophyll. After an interesting discussion, in which Sir D. Brandis, Mr. H. N. Ridley, and others took part, the Society adjourned to January 17.

PARIS.

Academy of Sciences, December 31, 1894.—M. Lœwy in the chair.—A study of graphites from iron, by M. Henri Moissan. At the ordinary pressure, the graphite is purer when formed at a higher temperature. The graphite produced at the highest temperature is the most stable in presence of nitric acid and potassium chlorate. Under pressure, the crystals and masses of graphite appear to have suffered incipient fusion. During the solution of the cast-iron by acids, hydroxy compounds are formed, which resist a dull red heat, but burn like the graphite itself.—Report was made favourably on a memoir by M. Riquier, to be printed in the *Recueil des Mémoires des Savants étrangers*, under the title "On the existence of integrals in any differential system, and on the reduction of such a system to a completely integrable linear form of the first order."—On the radial velocity of ζ Hercules, by M. H. Deslandres. A spectrophotographic determination in which the line displacements are measured on photographs taken with comparison spectra on each side of the star spectrum. The mean value of the radial velocity of this star is -70 km., as determined by M. Belopolsky; the author confirms this exceptional value, finding -60 41, a second observer finds the velocity -62 97.—On the determination of the number of roots common to a system of simultaneous equations, and on the calculation of the sum of the values of a function in these points, by M. Walther Dyck.—On the solution of numerical equations by means of recurring series, by M. R. Perrin.—On definite integrals of divisors, by M. N. Bougaïef.—On certain conditions to be realised for the measurement of electrical resistances by means of alternating currents and the telephone, by M. R. Colson.—On the sulphides of nickel and cobalt, by M. A. Villiers. This is a study of the conditions of precipitation of nickel and cobalt sulphides, and of the means whereby the precipitation may be wholly or partly prevented.—On calcium ethoxide, by M. de Forcrand. Calcium carbide, C_2Ca, gives with ethyl alcohol compounds of the type $n\,CaO + n'\,C_2H_6O$, and not an ethoxide. The compounds obtained are (1) $3CaO.4C_2H_6O$ and (2) $CaO.C_2H_6O$. At the same time gas is disengaged, consisting chiefly of acetylene (80 per cent.), and an easily liquefied ethylenic hydrocarbon (10 per cent.).—On β-oxycinchonine, by MM. E. Jungfleisch and E. Léger. The preparation and properties of this compound are given in detail, and thirteen of its salts are described. β-oxycinchonine is an energetic diacid base; it turns litmus blue, and reddens phenolphthalein.—Action of chlorine on the secondary alcohols, by M. A. Brochet. The action of chlorine on alcohols of the type $R.CHOH.CH_3$ yields ketones of the form $R.CO.CCl_3$, the radical R being also chlorinated to the extent characteristic of the radical.—On the industrial preparation and physiological properties of the oxalate and of the crystallised salts of nicotine, by MM. H. Parenty and E. Grasset. The mortal dose of pure nicotine is 20-21 mgm. per kilogram weight of animal. The fatal dose in the combined state is much larger. An animal slowly accustomed to the action of the poison can support daily a dose greater than that fatal in ordinary cases.—On pine tar, by M. Adolphe Renard.—Remarks on the muscles and bones of the hind limb of *Hatteria punctata*, by M. A. Perrin.—Comparative study of the lobed and reticulated Rhizopoda of fresh water, by M. Félix Le Dantec.—On the nests of *Vespa crabro*, L.; order of appearance of the earlier cells, by M. Charles Janet.—The fragments of the uppermost stratum of Ubaye, by MM. E. Haug and W. Kilian.—On the conditions of propagation of typhoid fever, cholera, and exanthematous typhus, by M. Renard.

NEW SOUTH WALES.

Linnean Society, November 28, 1894.—The President, Prof. David, in the chair.—Re-description of *Aspidites ramsavi*, Macl., by Edgar R. Waite.—A review of the fossil jaws of the *Macropodidæ* in the Queensland Museum., by C. W. De Vis. The very fine collection of over eleven hundred dissociated jaws or portions of jaws in the Queensland Museum has been studied in the light of a knowledge of the nature and range of the variations, individual and specific, presented by the skulls of 479 individuals referable to sixteen existing species. The following species were described as new: *Palorchestes parvus*, *Sthenurus pales*, *S. oreas*. *Halmaturus vinceus*, *H. thor*, *H. dryas*, *H. odin*, *H. indra*, *H. siva*, *H. vishnu*, *Macropus magister*, *M. pan*, and *M. faunus*.—Notes on some Land Planarians collected by Thos. Steel, Esq., on the Blue Mountains, N.S.W., by Dr. A. Dendy.—On a British bivalve mollusc found in Australia and Tasmania, with its distribution; and on a new sub-genus of *Trochidæ*, by J. Brazier.—*Cryptodon flexuosa*, Montagu, was recorded for the first time from Port Stephens, N.S.W., and Esperance Bay, Tasmania.—The name *Solanderia*, Fischer (1880), being preoccupied by Duchassaing and Michelotti (1846), it was proposed to replace it by *Rossiteria*.—Description of a new Australian eel, by J. Douglas Ogilby.—*Gymnothorax prionodon*, sp.n., from Port Jackson, appeared to be closely allied to the Atlantic species *G ocellatus*.—On a new Typhlops previously confounded with *T. unguirostris*, Peters, by G. A. Boulenger, F.R.S.—Botanical notes from the Technological Museum (part iii.), by J. H. Maiden and R. T. Baker. The writers gave a list, with localities, of plants new to New South Wales; also notes on some rare or little known plants indigenous to the colony.—On a new species of *Enteropneusta* from the coast of New South Wales, by James P. Hill. The name *Ptychodera australiensis* was proposed for the first described Australian species of Enteropneusta. It is specially characterised externally by the great development of the genital wings which completely hide the gill-area, and extend far into the hepatic region, and by the presence of two longitudinal epidermal stripes overlying the two ciliated bands of the intestine. In the mode of formation of the proboscis pore, it appears to be the most variable of all *Enteropneusta* hitherto described. The most interesting points in its internal anatomy are the presence of a median longitudinal infolding of the ventral wall of the heart bladder into the cavity of the same, the presence of a transverse vessel between the different proboscis vessels, and the much branched condition of the gonads.—On a Platypus embryo from the intrauterine egg, by J. P. Hill and C. J. Martin. The embryo described was taken from one of two eggs just ready to be laid. The eggs measured 18 mm. by 13·5—being somewhat larger than the eggs described by Caldwell. The embryo more nearly resembled that of the Virginian opossum (Didelphys) of seventy-three hours, described by Selenka, than any other embryo known to the authors. The Platypus embryo is, however, much longer.

BOOKS, PAMPHLET, and SERIALS RECEIVED.

BOOKS.—A Monograph of the Mycetozoa: A. Lister (London, Brit. Mus.).—Reisen in den Molukken: Prof. K. Martin, 2 Vols. Text and Plates (Leiden, Brill).—Harvard College, by an Oxonian: Dr. G. B. Hill (Macmillan).—Annuaire, 1895, par le Bureau des Longitudes (Paris Gauthier-Villars).—La Fabrication des Eaux-de-Vie: L. Jacquet (Paris, Gauthier-Villars).

PAMPHLET.—An Essay on Southerly Bursters: H. A. Hunt.

SERIALS.—Natural Science, January (Rait).—Astronomy & Astro-Physics, December (Wesley).—Journal of the Royal Microscopical Society, December (Williams).—Bulletin de L'Académie Impériale des Sciences de St. Pétersbourg, September, October, November (St Pétersbourg).—Bulletin of the American Mathematical Society, December (New York, Macmillan).—Humanitarian, January (Hutchinson).—Century Magazine, January (Unwin).—Contemporary Review, January (Isbister).—Natural History of Plants, Part 9: Kerner and Oliver (Blackie).—Fortnightly Review, January (Chapman).—Séances de la Société Française de Physique, 1894, 3e fasc. (Paris).—School Review, December (Hamilton, New York).—Zeitschrift für Physikalische Chemie, xv. Band, 4 Heft (Leipzig, Engelmann).—New Review, January (Heinemann).—Bulletin de la Société de Géographie, 3e Trim. 1894 (Paris).—Geological Magazine, January (Dulau).—Mind, January (Williams).—National Review, January (Arnold).—Cornhill Magazine, January (Smith, Elder).—Journal of the Royal Agricultural Society of England, Third Series, Vol. v. Part 4, No. 20 (Murray).—Annals of Scottish Natural History, January (Edinburgh, Douglas).—Internationales Archiv für Ethnographie, Band vii. Heft 5 and 6, and Supplement zu Band vii. (Leiden, Brill).—Quarterly Journal of Microscopical Science, December (Churchill).—Essex Naturalist, Nos. 6-10, Vol. viii. (Chelmsford, Durrant).—Scribner's Magazine, January (S. Low).—Phonographic Quarterly Review, No. 2 (Pitman).—Memoirs and Proceedings of the Manchester Literary and Philosophical Society, Fourth Series, Vol. 8, No. 4 (Manchester).—Science Progress, January (Scientific Press).—Round the World, Part 1 (Newnes).—Medical Magazine, January (140 Strand).

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